Winslow Township School District

Mathematics Curriculum – Algebra 2

Unit 3

<table>
<tr><th>Overview</th><th colspan="2">Standards for Mathematical Content</th><th>Unit Focus</th><th>Standards for Mathematical Practice</th></tr>
<tr><td>Unit 3

Periodic Models and the Unit Circle</td><td>● F.TF.A.1
● F.TF.A.2
● F.IF.C.7
● F.IF.B.4
● F.TF.B.5
● F.TF.C.8</td><td>● S.ID.B.6
● F.IF.C.9
● F.BF.A.1
● N.Q.A.2
● F.BF.B.3
● F.BF.B.4</td><td>● Extend the domain of trigonometric functions using the unit circle
● Analyze functions using different representations
● Interpret functions that arise in applications in terms of the context
● Model periodic phenomena with trigonometric functions
● Prove and apply trigonometric identities
● Summarize, represent, and interpret data on two categorical and quantitative variables
● Build a function that models a relationship between two quantities
● Build new functions from existing functions</td><td rowspan="2">MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments & critique the reasoning of others.

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

MP.8 Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Unit 3:

Suggested Open Educational Resources</td><td colspan="2">F.TF.A.1 Bicycle Wheel

F.TF.A.2 What exactly is a radian?

F.TF.A.2 Trigonometric functions for arbitrary angles (radians)

F.TF.A.2 Trig Functions and the Unit Circle

F.IF.B.4, F.IF.C.7e Model air plane acrobatics

F.TF.B.5 As the Wheel Turns

F.TF.C.8 Trigonometric Ratios and the Pythagorean Theorem</td><td>F.IF.C.9 Throwing Baseballs

F.BF.A.1b A Sum of Functions

F.BF.B.3 Exploring Sinusoidal Functions

F.BF.B.3 Transforming the graph of a function

F.BF.B.4a Temperatures in degrees Fahrenheit and Celsius</td></tr>
</table>

Major Supporting Additional (Identified by PARCC Model Content Frameworks).

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Curriculum Unit 3	Standards		Pacing	
			Days	Unit Days
Unit 3 **Periodic Models and the Unit Circle**	• F.TF.A.1 • F.TF.A.2 • F.TF.B.5 • F.TF.C.8 • F.BF.B.3 • F.BF.B.4	Use the radian measure of an angle to find the length of the arc in the unit circle subtended by the angle and find the measure of the angle given the length of the arc. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. Use the Pythagorean identity $(\sin\theta)^2 + (\cos\theta)^2 = 1$ to find $\sin\theta$, $\cos\theta$, or $\tan\theta$, given $\sin\theta$, $\cos\theta$, or $\tan\theta$, and the quadrant of the angle. Identify the effect on the graph of a polynomial, exponential, logarithmic, or trigonometric function of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative). Find the value of k given the graphs and identify even and odd functions from graphs and equations. Determine the inverse function for a simple function.	13	45
	• F.IF.C.7 • S.ID.B.6 • F.IF.C.9 • N.Q.A.2	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Represent nonlinear (exponential and trigonometric) data for two variables on a scatter plot, fit a function to the data, analyze. Analyze and compare properties of two functions when each is represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Define appropriate quantities for the purpose of descriptive modeling.	12	
	• F.IF.B.4 • F.BF.A.1	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Write a function that describes a relationship between two quantities.	15	
	Assessment, Re-teach and Extension		5	

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Unit 3 Algebra 2		
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
● F.TF.A.1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. ● F.TF.A.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	MP.3 Construct viable arguments and critique the reasoning of others. MP.6 Attend to precision.	Concepts: ● Radian measure of an angle as the length of the arc on the unit circle that is subtended by the angle ● Relationship between degrees and radians Students are able to: ● find the measure of the angle given the length of the arc. ● find the length of an arc given the measure of the central angle. ● convert between radians and degrees. ● use the unit circle to evaluate sine, cosine and tangent of standard reference angles. Learning Goal 1: Use the radian measure of an angle to find the length of the arc in the unit circle subtended by the angle and find the measure of the angle given the length of the arc. Learning Goal 2: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
● F.IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.C.7e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. ● F.IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include:*	MP.1 Make sense of problems and persevere in solving them. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure.	Concepts: ● Relationship between the unit circle in the coordinate plane and graph of trigonometric functions. Students are able to: ● graph trigonometric functions, showing period, midline, and amplitude. Learning Goal 3: Graph trigonometric functions expressed symbolically, showing key features of the graph, by hand in simple cases and using technology for more complicated cases.

intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.		
• F.TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	MP.4 Model with mathematics.	Concepts: • Periodic functions may model real-world scenarios. Students are able to: • use characteristics of real world phenomena to select a trigonometric model. • identify amplitude, frequency and midline appropriate for the model. Learning Goal 4: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
• F.TF.C.8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$,or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.	MP.3 Construct viable arguments and critique the reasoning of others. MP.5 Use appropriate tools strategically. MP.7 Look for and make use of structure.	Concepts: No new concept(s) introduced Students are able to: • prove the Pythagorean identity: $\sin^2(\theta) + \cos^2(\theta) = 1$. • use the Pythagorean identity to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ when given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. Learning Goal 5: Use the Pythagorean identity $(\sin\theta)^2 + (\cos\theta)^2 = 1$ to find $\sin\theta$, $\cos\theta$, or $\tan\theta$, given $\sin\theta$, $\cos\theta$, or $\tan\theta$, and the quadrant of the angle.
• S.ID.B.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related 6a. Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision.	Concepts: No new concept(s) introduced Students are able to: • fit exponential and trigonometric functions to data using technology. • solve problems using functions fitted to data (prediction equations). • interpret the intercepts of models in context. • Plot residuals of non-linear functions. • Analyze residuals in order to informally evaluate the fit of exponential and trigonometric functions. Learning Goal 6: Represent nonlinear (exponential and trigonometric) data for two variables on a scatter plot, fit a function to the data, analyze residuals (in order to informally assess fit), and use the function to solve problems. Use given functions or choose a function suggested by the context; emphasize exponential and trigonometric models.

• F.IF.C.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	MP.1 Make sense of problems and persevere in solving them. MP.3 Construct viable arguments and critique the reasoning of others. MP.5 Use appropriate tools strategically. MP.6 Attend to precision.	Concepts: No new concept(s) introduced Students are able to: • compare key attributes of functions each represented in a different way (i.e zeros, end behavior, periodicity, asymptotes). Learning Goal 7: Analyze and compare properties of two functions when each is represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
• F.BF.A.1. Write a function that describes a relationship between two quantities. F.BF.A.1b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* • N.Q.A.2. Define appropriate quantities for the purpose of descriptive modeling.	MP.4 Model with mathematics. MP.7 Look for and make use of structure.	Concepts: • Functions of various types can be combined to model real world situations. Students are able to: • use arithmetic operations to combine functions of varying types in order to model relationships between quantities. Learning Goal 8: Construct a function that combines, using arithmetic operations, standard function types to model a relationship between two quantities.
• F.BF.B.3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	MP.3 Construct viable arguments and critique the reasoning of others. MP.5 Use appropriate tools strategically. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.	Concepts: • Function notation representation of transformations Students are able to: • perform transformations on graphs of polynomial, exponential, logarithmic, or trigonometric functions. • identify the effect on the graph of replacing f(x) by – $f(x) + k$; – $k\,f(x)$; – $f(kx)$; – and $f(x + k)$ for specific values of k (both positive and negative). • identify the effect on the graph of combinations of transformations. • given the graph, find the value of k. • illustrate an explanation of the effects on polynomial, exponential, logarithmic, or trigonometric graphs using technology. Learning Goal 9: Identify the effect on the graph of a polynomial, exponential, logarithmic, or trigonometric function of replacing f(x) by f(x) +

		k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative). Find the value of k given the graphs and identify even and odd functions from graphs and equations.
• F.BF.B.4. Find inverse functions. F.BF.B.4a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x^3 or f(x) = (x+1)/(x–1) for x ≠1.* [*note: composition of functions is not introduced here]	MP.1 Make sense of problems and persevere in solving them. MP.6 Attend to precision. MP.8 Look for and express regularity in repeated reasoning.	Concepts: • For a function f(x) that has an inverse, the domain/input for f(x) is the inverse function's range/output and that the range/output for f(x) is the inverse function's domain/input. Students are able to: • use function notation to represent the inverse of a function – $f^{-1}(x)$. • transform an equation in order to isolate the independent variable, recognizing that the domain/input for f(x) is the inverse function's range/output and that the range/output for f(x) is the inverse function's domain/input. Learning Goal 10: Determine the inverse function for a simple function.

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District/School Formative Assessment Plan	**District/School Summative Assessment Plan**
Pre-Assessment, Quizzes Exit Tickets Daily Monitoring Linkit!	Unit Benchmark Linkit! Diagnostic

Focus Mathematical Concepts

Prerequisite skills:

Students should be able to:

- understand that angle measures in radians may be determined by a ratio of intercepted arc to radius
- have the ability to convert between degree and radian measure
- have the ability to connect knowledge of special right triangles gained in Geometry to evaluating trigonometric functions at any domain value
- have the ability to recognize graphs of parent functions of trigonometric functions
- have the ability to connect contextual situations to appropriate trigonometric functions: e.g. using sine or cosine to model cyclical behavior

Common Misconceptions:

Students may confuse the concepts of reference angle and co-terminal angles.

Students may believe that it is reasonable to input any *x*-value into a function, so they will need to examine multiple situations in which there are various limitations to the domains.

Students may also believe that the slope of a linear function is merely a number used to sketch the graph of the line. In reality, slopes have real-world meaning, and the idea of a rate of change is fundamental to understanding major concepts from geometry to calculus.

Students may believe that the best (or only) way to generalize a table of data is by using a recursive formula.

Students naturally tend to look "down" a table to find the pattern but need to realize that finding the 100^{th} term requires knowing the 99^{th} term unless an explicit formula is developed.

Students may also believe that arithmetic and geometric sequences are the same. Students need experiences with both types of sequences to be able to recognize the difference and more readily develop formulas to describe them.

Fluency Recommendations:

A-SSE.A.2 The ability to see structure in expressions and to use this structure to rewrite expressions is a key skill in everything from advanced factoring (e.g., grouping) to summing series to the rewriting of rational expressions to examine the end behavior of the corresponding rational function.

F-IF.A.3 Fluency in translating between recursive definitions and closed forms is helpful when dealing with many problems involving sequences and series, with applications ranging from fitting functions to tables to problems in finance.

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District/School Tasks	District/School Primary and Supplementary Resources and Technology Integration
PARCC Released Items **http://www.parcc-assessment.org/released-items** **NJDOE Digital Item Library** **https://nj.digitalitemlibrary.com/home** **NJSLA Mathematics Evidence Statements** **https://docs.google.com/spreadsheets/d/18M5r1jk4P729fTpAlWAzrw1gE6tken233I-Yk0U712M/edit#gid=554025491** **LinkIt! Form A, B, & C**	**Textbook** **IXL** https://www.ixl.com/ **Khan Academy** https://www.khanacademy.org/ **HS Flip Book:** http://community.ksde.org/Default.aspx?tabid=5646 **North Carolina Dept of Ed. Wikispaces:** http://maccss.ncdpi.wikispaces.net/High+School **101 Math Discourse Questions:** http://www.casamples.com/downloads/100MathDiscourseQuestions_Printable.pdf **Asking Effective Questions** http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/CBS_AskingEffectiveQuestions.pdf **Diversity, Equity & Inclusion Educational Resources** https://www.nj.gov/education/standards/dei/
Instructional Best Practices and Exemplars	
1. Identifying similarities and differences 2. Summarizing and note taking 3. Reinforcing effort and procing recognition 4. Homework and practice 5. Nonlinguistic representations	6. Cooperative learning 7. Setting objectives and providing feedback 8. Generating and testing hypotheses 9. Cues, questions, and advance organizers 10. Manage response rate

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Vocabulary			
absolute value function complex numbers complex roots function	exponential inverse function Laws of Logarithms	relative maximums relative minimums Step function	symmetries transformations trigonometric

9.1 Personal Financial Literacy, 9.2 Career Awareness, Exploration, Preparation and Training & 9.4 Life Literacies and Key Skills

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
9.4.12.TL.3: Analyze the effectiveness of the process and quality of collaborative environments.

The implementation of the 21st Century skills and standards for students of the Winslow Township District is infused in an interdisciplinary format in a variety of curriculum areas that include, English language Arts, Mathematics, School Guidance, Social Studies, Technology, Visual and Performing Arts, Science, Physical Education and Health, and World Language.
Additional opportunities to address 9.1, 9.2 & 9.4:

Philadelphia Mint
https://www.usmint.gov/learn/kids/resources/educational-standards

Different ways to teach Financial Literacy.
https://www.makeuseof.com/tag/10-interactive-financial-websites-teach-kids-money-management-skills/

Suggested Modifications for Special Education/504

Students with special needs: The students' needs will be addressed on an individual and grade level using a variety of modalities. Accommodations will be made for those students who need extra time to complete assignments. Support staff will be available to aid students related to IEP specifications. 504 accommodations will also be attended to by all instructional leaders. Physical expectations and modifications, alternative assessments, and scaffolding strategies will be used to support this learning. The use of Universal Design for Learning (UDL) will be considered for all students as teaching strategies are considered.

☐ Provide the opportunity to re-take tests
☐Modify activities/assignments/projects/assessments
☐ Breakdown activities/assignments/projects/assessments into manageable units
☐Additional time to complete activities/assignments/projects/assessments
☐ Provide an option for alternative activities/assignments/projects/assessments
☐ Modify Content
☐ Modify Amount
☐ Small Group Intervention/Remediation
☐ Individual Intervention/Remediation
☐ Additional Support Materials
☐ Guided Notes
☐ Graphic Organizers
☐ Adjust Pacing of Content
☐ Increase one on one time
☐ Peer Support
☐ Other Modifications for Special Education:

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<table>
<tr><th colspan="2">Suggested Modifications for At-Risk Students</th></tr>
<tr><td colspan="2">Formative and summative data will be used to monitor student success. At first signs of failure, student work will be reviewed to determine support. This may include parent consultation, basic skills review and differentiation strategies. With considerations to UDL, time may be a factor in overcoming developmental considerations
☐ Provide the opportunity to re-take tests
☐ Increase one on one time
☐ Oral prompts can be given
☐ Using visual demonstrations, illustrations, and models
☐ Give directions/instructions verbally and in simple written format
☐ Peer Support
☐ Modify activities/assignments/projects/assessments
☐ Additional time to complete activities/assignments/projects/assessments
☐ Provide an option for alternative activities/assignments/projects/assessments
☐ Modify Content
☐ Modify Amount
☐ Adjust Pacing of Content
☐ Small Group Intervention/Remediation
☐ Individual Intervention/Remediation
☐ Additional Support Materials
☐ Guided Notes
☐ Graphic Organizers
☐ Other Modifications for Students At-Risk:</td></tr>
<tr><th>Suggested for English Language Learners</th><th>Suggested Modifications for Gifted Students</th></tr>
<tr><td>All WIDA Can Do Descriptors can be found at this link: https://wida.wisc.edu/teach/can-do/descriptors
☐ Grades 9-12 WIDA Can Do Descriptors:
☐ Listening ☐ Speaking
☐ Reading ☐ Writing
☐ Oral Language
Students will be provided with accommodations and modifications that may include:
• Relate to and identify commonalities in mathematics studies in student's home country
• Assist with organization
• Use of computer
• Emphasize/highlight key concepts
• Teacher Modeling
• Peer Modeling
• Label Classroom Materials - Word Walls</td><td>Students excelling in mastery of standards will be challenged with complex, high level challenges related to the topic.
• Raise levels of intellectual demands
• Require higher order thinking, communication, and leadership skills
• Differentiate content, process, or product according to student's readiness, interests, and/or learning styles
• Provide higher level texts
• Expand use of open-ended, abstract questions
• Critical and creative thinking activities that provide an emphasis on research and in-depth study
• Enrichment Activities/Project-Based Learning/ Independent Study
Additional Strategies may be located at the links:
❖ Gifted Programming Standards
❖ Webb's Depth of Knowledge Levels and/or Revised Bloom's Taxonomy
❖ REVISED Bloom's Taxonomy Action Verbs</td></tr>
</table>

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Suggested Activities	
☐ Do Now/Warm-Up	☐ Centers
☐ Whole Group	☐ Intervention/Remediation
☐ Small Groups	☐ Projects
☐ Guided Practice	☐Academic Games
☐ Independent Practice	☐ Other Suggested Activities:

Interdisciplinary Connections

Big Ideas Real-Life STEM Videos and Performance Tasks

Interdisciplinary Connections: ELA

NJSLSA.R1. Read closely to determine what the text says explicitly and to make logical inferences and relevant connections from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.

NJSLSA.W2. Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content

NJSLSA.L1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking

SL.9-10.4: Present information, findings and supporting evidence clearly, concisely and logically. The content, organization, development and style are appropriate to task, purpose and audience.

NJSLSA.L6: Acquire and use accurately a range of general academic and domain-specific words and phrases sufficient for reading, writing, speaking and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when encountering an unknown term important to comprehension or expression.

Integration of Computer Science and Design Thinking NJSLS 8

8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original and existing algorithms.
8.1.12.AP.2: Create generalized computational solutions using collections instead of repeatedly using simple variables.
8.1.12.AP.8: Evaluate and refine computational artifacts to make them more usable and accessible.
8.2.12.ETW.2: Synthesize and analyze data collected to monitor the effects of a technological product or system on the environment. • 8.2.12.ETW.3: Identify a complex, global environmental or climate change issue, develop a systemic plan of investigation, and propose an innovative sustainable solution.
8.2.12.EC.3: Synthesize data, analyze trends, and draw conclusions regarding the effect of a technology on the individual, culture, society, and environment and share this information with the appropriate audience.